Game theory applying to military and trade wars

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Abstract: The military and economic wars are two major kinds of wars that affect the relationship between countries in recent years. Both kind of wars could change the status of one country, while military wars may seem to be a more direct and sharp way then economic wars. Game theory is widely used in the area of military wars and trade conflicts. Such application can be helpful with decision making. While for different situation, different strategy should be used. In this article, in order to give the optimal strategy in different scenarios, some basic ideas of game theory applying to military and economic conflicts is presented by reviewing different ideas given by scholars. It can be concluded that for both kind of wars, it is vital to observe the rivalry's decision and then have some adjustments. By giving such reviews, it could be helpful for government to make decision when they face with similar situations.

1. Introduction

The history of military war could date back to primitive society. There were several battles that changed the history, and it has always been regarded as a vital mean to change the world thus there were always people studying different aspects that would affect a war. With the development of military wars, people started to not only pay attention to those traditional concerns like the weapons and tactics, but also to the interactions between wars and society, trades etc. [1]. Actually, the relationship between trade and military wars are really close. It is being argued that trade has its natural effect to bring about peace. While, since 1870, the correlation between trade and wars became unclear, and even in some cases, there tended to be an opposite relation [2]. Nowadays, as economic development plays an increasingly vital role among the development of the country, it is more common to introduce trade war as a sanction method to limit the development of other countries.

The changes of wars bring about the fact that commanders should consider more about only military power. More research and scientific methods should be used to assist the decision making. Game theory as an analytic tool could be regarded as effective in both fields of military wars and trade wars when decision making is needed. Thomas Edison first applied game theory to real military events in 1917, and O.G. Haywood, Jr published his pioneering article considering the relations between military doctrines and game theory [3]. Such kind of logical analysis can help with the efficient allocation, building, weapons as well as better conduction of wars [4]. As a result of the similarity of trade wars and military wars, game theory has also been used to analyze different trade conflicts between countries.

This article is aiming to review examples in which game theory worked as a tool to assist decision-making in military wars and trade wars. It will also focus on the process of how it helped to find out the optimal strategy. Even though our article mainly focused on the simplest theories, but for those who want to deal with some less focused or more complicated situations, our article can provide inspirations and provide them with some basic ideas.

In the remains paper below, wars are divided into military conflicts and trade wars, for the first section, this article will view and analyze the war from the side of two-person zero-sum game and

two-person non-zero-sum game. In both kinds of games, different solutions are given to find the optimal strategy. While for the second section, the trade relations of the United States with different countries were reviewed in the perspective of game theory. The last section will be a conclusion of the whole paper.

2. Two-person zero-sum game and military conflicts

The simplest form of zero-sum game is the two-person zero-sum game, in which there is no chance for co-operation, instead, non-cooperation is more attractive with the presence of competitions [5]. In this section, all the wars given as examples will be presented from the prospective of two-person zero-sum games.

2.1Max-min strategy when facing two-player zero-sum games in military wars

A scholar applied concepts of game theory to a battle between America and Japan and try to find out the optimal strategy for the later country. A payoff matrix with maximin and minimax was proposed to make it easier to compare different payoffs by choosing different strategies [6]. By introducing two concepts, namely maximin (maximum of the minimums in the line) and minimax (the minimums of the maximums in the line), different players can choose their optimal option base on their doctrine. By applying game theory, the American army would choose northern route could have a promised outcome of 2-day-bombing. (The payoff matrix could be seen in table 1 and table 2.)

Table 1. payoff matrix [6]

		Japanese	
		Northern	Southern
American	Northern	2 days	2 days
	southern	1 day	3 days

Table 2. payoff matrix (with maximin and minimax) [6]

		Japanese		
		Northern	Southern	
American	Northern	2 days	2 days	2 days (maximin)
	southern	1 day	3 days	
		2 days (minimax)		

Some scholars reviewed two concepts announced by von Neuman which was ‘minorant game’ and ‘majorant game’ respectively, they based the examples on these two kinds of games and announced several limitations of these models when they were used in real life. Apart from that, methods to make the aid more accurate are also mentioned. As a result, the effectiveness of the application of max-min strategy in military wars could be better promised. [7]

2.2 Matched strategies when facing two-player zero-sum games in military wars

Another strategy was being used in the war showed in this section.

In order to find out how optimal choice are decided based on rivalry’s decision in military wars, a payoff matrix without visualized outcome was showed. [6] In this game, the American commander based his choice on his opposing side’s choice. Assuming the opposing army would choose to withdraw, the U.S commander then chose the best choice among all the least favorable choices. They eventually chose to concentrate their reconnaissance craft on northern route.

Table 3. payoff matrix for the six strategies

		German		Minimum of row
		Attack gap	Withdraw	
U.S	Reinforce gap	Gap holds	Weak pressure on German withdrawal	Gap holds
	Move reserve eastward	Gap cut	Strong pressure on German withdrawal	Gap cut
	Hold reserve in place one day	Gap holds possibly, Germans circled	Moderate pressure on German withdrawal	Moderate pressure on German withdrawal
Maximum of column		Gap holds	Strong pressure on German withdrawal	

In order to weigh the feasibility of relying decisions on the others' possible choice in military conflicts, different scenarios for two players in a simplified two-person zero-sum game where only two strategies are available was listed in the paper [8]. It was then being concluded that for most of the times, getting know more about the army's intention would not be useful for better decision as a result of the over simplification of the model we used.

For many conflicts, the pressures given by time could not be ignored. Considering the deadlines, a commonly used ad hoc strategy also based on the rivalry's option was mentioned in the article [9]. After estimating the choice of the rivalry, the choice chosen would be opposite to both the choices of two sides in order to make sure the optimum strategy would involve a conflict. It was argued that doing this would bring about an arbitrary payoff function.

3. Two-person nonzero-sum game and military conflicts

Comparing to zero-sum games, the nonzero-sum games have richer set of features as a result of the involvement of the interaction of information structures and existence of equilibria [10]. For example, there may be differentiated possible strategies for players in such kind of game depending on the ability of the players 'access to information, possible strategies include minimax, noninferior and Nash equilibrium [11] (a situation where all of the players 'choices are in the best response of other players, and none of them are willing to change their choice [12]).

In this section, all the wars given as examples will be presented from the prospective of two-person nonzero-sum games.

3.1 Military wars analysis from the perspective of game theory

In the cold war, scholars suggested that the opposing alliances of NATO and Warsaw Pact were in a prisoners' dilemma (a situation where all the players would eventually choose the strategy that bring them the worst outcome [13]) [9]. To explain the final choice of both sides, the article considered the outcome when both chose to arm or not arm and when one chose to arm and the other chose to not arm respectively, and concluded that though involving more costs, both sides would choose to arm.

Sometimes, facing with different difficulties [14], it is possible for the two players in a prisoner dilemma to cooperate, after all, it was not possible in this war.

To explain the reason why cooperation was not available in the cold war, the article applied the concept of prisoner's dilemma and Nash equilibrium to arm race [12]. As a result, it was being argued that cooperation cannot be regarded as an equilibrium since it was instable, thus there were difficulties of maintaining the cooperation. Therefore, cooperation cannot be a solution to this game.

Different numbers have been given to the side of the United States and Soviet Union engaging in the Cuban missile crisis to represent the payoffs in the article [15]. By considering the problems of simply applying the 'chicken' to such situation and neither side wanted to make an all-or-nothing choice, 'alternative' as a strategy not only base the decision on the basic concept of Nash equilibrium was being stated and being regarded as a more feasible strategy to such case.

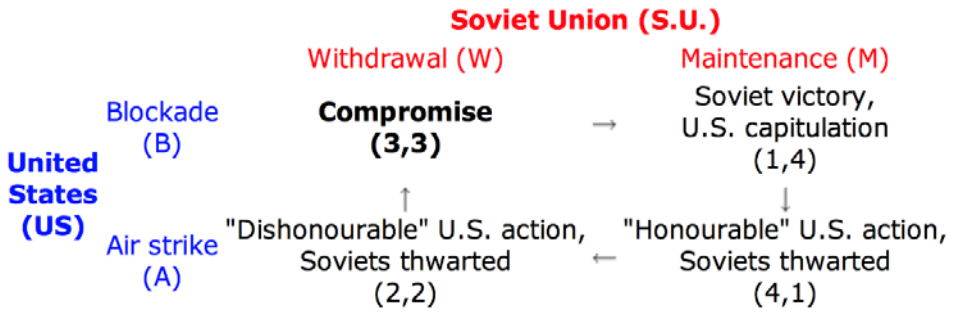

Figure1. Cuban missile crisis as Alternative [15]

4. Some trade wars

4.1 Trade war among the European countries and other countries

Several articles talked about the issue happened between US and Western countries. The trade friction between EC and the United States mainly resulted from the differences and conflicts between the two sides in the protection of internal key industries, internal policies, preferential trade system and the concept of fair trade. [16]

An essay considered the value of trade negotiations in a trade war by giving the example of non-cooperative game between the United States and EC. The article assumed that two pure strategy which was keep the original structure of protection in place, or to abolish trade barriers respectively. It turned out that the US's decision cannot be decided without knowing what EU did. The US could only decide an optimal choice after EC took a step. [17]

In order to find the implications of trade disputes among European countries and PRC, some scholars used data of the payoff of different chosen strategies in a bilateral trade and then developed a zero-sum trade war game. As a result, they found that to maximize the profit of the European countries, no tariff should be imposed on the products of PRC. [18]

4.2 Trade wars between America and Japan

This section would review the trade relation between Japan and US. In recent years, the US and Japan's trade relation have changed a lot from simple bilateral competition to global competition of FDI (foreign direct investment) [19]. Reviewing previous trade wars from the perspective of game theory could be helpful when analyzing the economic policies of other countries.

To figure out how decision was made in trade wars, scholars reviewed the usage of game theory in several means that could be used in trade wars including methods of economic sanctions, the formation of different kind of economic organizations and economic policies etc. [20]. It was stated that a strategic interdependence existed in all listed above. As a result, it was concluded that the decision of one country in a trade interaction would depend on what the other country did. And this was same to what was argued in game theory.

To figure out an issue that had threatened the US antidumping duties, a three-country oligopoly model was used to analyze cyclical dumping during the negotiation of the 1986 Arrangement between Japan and America. In this event, they were rejected by negotiating parties in favor of price floors. However, the Japanese Government protected itself was by making use of its status as the dominant supplier; it mandated cuts in exports and raised prices. [21]

4.3 Trade war between America and China

In this section, another example about the relation between China and America will be reviewed. They are both two largest economy system in the world and each are the other's largest trading partners. It was stated by Morison that The U.S.-China trade relationship is critical. Contemporarily, their trade relationship was mutually beneficial to both sides, making their economies connected [22].

Since the trade war between US and China didn't end, one paper constructed a game theory model aiming to find out the strategies that could be used by the two countries. A pure strategy payoff matrix

and a mixed strategy payoff matrix was present respectively and it was said that a Nash equilibrium only existed in the later scenario. As a result, it was announced that neither country had a dominant strategy that would prevent them from losing, both of them should better estimate the move of their rivalry and then decide what to do next. [23]

Table 4. payoff matrix of pure strategy [33]

		US	
		Trade protection	Free trade
CHN	Trade protection	X_2+D-F, X_1+B-E	X_2+D, X_1-C
	Free trade	X_2-A, X_1+B	X_2, X_1

Table 5. payoff matrix of mixed strategy [23]

		US	
		Trade protection(p)	Free trade(1-p)
CHN	Trade protection (q)	X_2+D-F, X_1+B-E	X_2+D, X_1-C
	Free trade(1-q)	X_2-A, X_1+B	X_2, X_1

In order to analyze the trade conflicts between China and the US, by using the game theory models which shown those payoff and strategies, the scholar found that several major trade issues became a flashpoint for a trade war and US would choose to "fight" with China. In the year of 2007, China became the second largest economy in the world based on GDP, and the US was then faced with a large challenge. After considering the payoff of different trading strategy and to maintain their economic status, they eventually chose to decrease the trade cooperation with China. [24]

Jason Z. Yin used nonzero-sum game that modeled international trade and cooperation between countries as a two-player game with two strategies to show different influences of a trade war between the US and China and discovered the impact of protectionism and retaliatory protectionism. The article found that this made the trade relation between US and China more tense. The essay also considered a trade war as a simple game and examine its outcomes, altering payoffs in the trade war outcome to correspond with GDP growth and welfare losses. By shifting to defection strategies to punish the other player for defection, both countries would enforce cooperation in the relationship [25].

Table 6. payoff matrix of China and US [25]

<table>
<tr><td>China
US</td><td>Cooperate</td><td colspan="3">Defect</td></tr>
<tr><td>Cooperate</td><td>4,4</td><td colspan="3">0,3</td></tr>
<tr><td rowspan="3">Defect</td><td rowspan="3">3,0</td><td>China
US</td><td>Turn</td><td>stay</td></tr>
<tr><td>turn</td><td>0,0</td><td>-1,1</td></tr>
<tr><td>stay</td><td>1,-1</td><td>-10,-10</td></tr>
</table>

5. Conclusion

How game theory was applied in military wars and trade wars have been reviewed in this paper. Game theory can be regarded as helpful when it comes to decision-making in both military wars and trade wars. Such theory has been widely and successfully used when people being faced with military and trade wars and can promote a better achievement of the goal and mission. In this paper, a brief introduction of game theory is being given, then it comes to the military war examples of two-person zero-sum games and two-person nonzero-sum games, different ways of finding the optimal strategy in these games are being reviewed. Among these researches, max-min strategy and matched strategy are the two mainly used strategies in military wars. And in trade conflicts, the choice of the rivalry is one of the most vital influencing factors. It can be concluded that in both kind of wars, the interrelationship of the choices of both sides played an incredibly important role in decision making

and should always be carefully considered. The contribution of this paper in the application of game theory to military wars and trade wars can effectively help researchers in the area of decision making in such occasions.

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